1 ●次の関数を微分せよ。

(1) $y=x^7$

$y'=7x^6$

(2) $y=-5x^6$

$y'=-5\cdot 6x^5=-30x^5$

(3) $y=x^5-2x^3$

$y'=5x^4-2\cdot 3x^2=5x^4-6x^2$

(4) $y=2x^5+x^4+6x^3+7x^2+8x+3$

$y'=2\cdot 5x^4+4x^3+6\cdot 3x^2+7\cdot 2x+8$

$=10x^4+4x^3+18x^2+14x+8$

(5) $y=\frac{1}{3}x^6-x^5+\frac{3}{2}x^4-4x^3-9x$

$y'=\frac{1}{3}\cdot 6x^5-5x^4+\frac{3}{2}\cdot 4x^3-4\cdot 3x^2-9$

$=2x^5-5x^4+6x^3-12x^2-9$

(6) $y=x^{-3}$

$y'=-3x^{-3-1}=-3x^{-4}\left(=-\frac{3}{x^4}\right)$

(7) $y=-4x^{-5}$

$y'=-4(-5x^{-5-1})=20x^{-6}\left(=\frac{20}{x^6}\right)$

(8) $y=\frac{1}{x^7}$

$y=\frac{1}{x^7}=x^{-7}$

よって　$y'=-7x^{-7-1}=-7x^{-8}=-\frac{7}{x^8}$

(9) $y=x^4+\frac{1}{x^3}$

$y=x^4+\frac{1}{x^3}=x^4+x^{-3}$

よって　$y'=4x^3-3x^{-3-1}=4x^3-3x^{-4}$

$=4x^3-\frac{3}{x^4}$

(10) $y=\frac{2x^3-x^2+3}{x^2}$

$y=\frac{2x^3-x^2+3}{x^2}=2x-1+\frac{3}{x^2}$

$=2x-1+3x^{-2}$

よって　$y'=2+3(-2x^{-2-1})=2-6x^{-3}$

$=2-\frac{6}{x^3}=\frac{2x^3-6}{x^3}$

2 ●次の関数を微分せよ。

(1) $y=(x^2-2x+5)^2$

$y'=2(x^2-2x+5)(x^2-2x+5)'$

$=2(x^2-2x+5)(2x-2)$

$=4(x-1)(x^2-2x+5)$

(2) $y=-(3x^3+x+1)^4$

$y'=-4(3x^3+x+1)^3(3x^3+x+1)'$

$=-4(3x^3+x+1)^3(9x^2+1)$

(3) $y=(2x+1)(4x-3)^2$

$y'=(2x+1)'(4x-3)^2+(2x+1)\{(4x-3)^2\}'$

$=2(4x-3)^2+(2x+1)\cdot 2(4x-3)(4x-3)'$

$=2(4x-3)^2+(2x+1)\cdot 2(4x-3)\cdot 4$

$=2(4x-3)\{(4x-3)+4(2x+1)\}$

$=2(4x-3)(12x+1)$

(4) $y=\frac{1}{(5x^2-x-1)^4}$

$y=\frac{1}{(5x^2-x-1)^4}=(5x^2-x-1)^{-4}$

よって　$y'=-4(5x^2-x-1)^{-5}(5x^2-x-1)'$

$=-4(5x^2-x-1)^{-5}(10x-1)$

$=-\frac{4(10x-1)}{(5x^2-x-1)^5}$

(5) $y=\left(x^2-\frac{2}{x}\right)^2$

$y'=2\left(x^2-\frac{2}{x}\right)\left(x^2-\frac{2}{x}\right)'$

$=2\left(x^2-\frac{2}{x}\right)\left(2x+\frac{2}{x^2}\right)$

$=4\left(x^2-\frac{2}{x}\right)\left(x+\frac{1}{x^2}\right)$

3 ●次の関数を微分せよ。

(1) $y=3(x^3+4)^2$

$y'=3\cdot 2(x^3+4)(x^3+4)'$

$=6(x^3+4)\cdot 3x^2$

$=18x^2(x^3+4)$

(2) $y=(2x^2-1)^5$

$y'=5(2x^2-1)^4(2x^2-1)'$

$=5(2x^2-1)^4\cdot 4x$

$=20x(2x^2-1)^4$

(3) $y=(x^2-1)(x^2+9)^2$

$y'=(x^2-1)'(x^2+9)^2+(x^2-1)\{(x^2+9)^2\}'$

$=2x(x^2+9)^2+(x^2-1)\cdot 2(x^2+9)(x^2+9)'$

$=2x(x^2+9)^2+(x^2-1)\cdot 2(x^2+9)\cdot 2x$

$=2x(x^2+9)\{(x^2+9)+2(x^2-1)\}$

$=2x(x^2+9)(3x^2+7)$

(4) $y=-\frac{1}{(x^2+3)^3}$

$y=-\frac{1}{(x^2+3)^3}=-(x^2+3)^{-3}$

よって　$y'=-\{-3(x^2+3)^{-4}(x^2+3)'\}$

$=3(x^2+3)^{-4}\cdot 2x$

$=\frac{6x}{(x^2+3)^4}$

(5) $y=\left(4x+\frac{1}{x^2}\right)^3$

$y'=3\left(4x+\frac{1}{x^2}\right)^2\left(4x+\frac{1}{x^2}\right)'$

$=3\left(4x+\frac{1}{x^2}\right)^2\left(4-\frac{2}{x^3}\right)$

$=6\left(4x+\frac{1}{x^2}\right)^2\left(2-\frac{1}{x^3}\right)$

4 ●次の関数を微分せよ。

(1) $y=x^{\frac{4}{7}}$

$y'=\frac{4}{7}x^{\frac{4}{7}-1}=\frac{4}{7}x^{-\frac{3}{7}}\left(=\frac{4}{7\sqrt[7]{x^3}}\right)$

(2) $y=\frac{1}{\sqrt{x^3}}$

$y=\frac{1}{\sqrt{x^3}}=x^{-\frac{3}{2}}$

よって　$y'=-\frac{3}{2}x^{-\frac{3}{2}-1}=-\frac{3}{2}x^{-\frac{5}{2}}$

$=-\frac{3}{2\sqrt{x^5}}\left(=-\frac{3}{2x^2\sqrt{x}}\right)$

(3) $y=x\sqrt[4]{x}$

$y=x\sqrt[4]{x}=x^{\frac{5}{4}}$

よって　$y'=\frac{5}{4}x^{\frac{5}{4}-1}=\frac{5}{4}x^{\frac{1}{4}}=\frac{5}{4}\sqrt[4]{x}$

(4) $y=4x^{\frac{3}{2}}-6x^{\frac{5}{6}}$

$y'=4\cdot\frac{3}{2}x^{\frac{3}{2}-1}-6\cdot\frac{5}{6}x^{\frac{5}{6}-1}=6x^{\frac{1}{2}}-5x^{-\frac{1}{6}}\left(=6\sqrt{x}-\frac{5}{\sqrt[6]{x}}\right)$

(5) $y=3x^2\cdot\sqrt[3]{x}-\frac{1}{x\sqrt[5]{x}}$

$y=3x^2\cdot\sqrt[3]{x}-\frac{1}{x\sqrt[5]{x}}=3x^{\frac{7}{3}}-x^{-\frac{6}{5}}$

よって　$y'=3\cdot\frac{7}{3}x^{\frac{7}{3}-1}-\left(-\frac{6}{5}x^{-\frac{6}{5}-1}\right)$

$=7x^{\frac{4}{3}}+\frac{6}{5}x^{-\frac{11}{5}}$

$=7x\sqrt[3]{x}+\frac{6}{5x^2\cdot\sqrt[5]{x}}$

5 ●次の関数を微分せよ。

(1) $y=-x^{\frac{6}{5}}$

$y'=-\frac{6}{5}x^{\frac{6}{5}-1}=-\frac{6}{5}x^{\frac{1}{5}}\left(=-\frac{6}{5}\sqrt[5]{x}\right)$

(2) $y=\frac{1}{\sqrt{x}}$

$y=\frac{1}{\sqrt{x}}=x^{-\frac{1}{2}}$

よって　$y'=-\frac{1}{2}x^{-\frac{1}{2}-1}=-\frac{1}{2}x^{-\frac{3}{2}}$

$=-\frac{1}{2\sqrt{x^3}}\left(=-\frac{1}{2x\sqrt{x}}\right)$

(3) $y=2x\sqrt[6]{x}$

$y=2x\sqrt[6]{x}=2x^{\frac{7}{6}}$

よって　$y'=2\cdot\frac{7}{6}x^{\frac{7}{6}-1}$

$=\frac{7}{3}x^{\frac{1}{6}}=\frac{7}{3}\sqrt[6]{x}$

(4) $y=3x^{\frac{4}{3}}+5x^{\frac{3}{8}}$

$y'=3\cdot\frac{4}{3}x^{\frac{4}{3}-1}+5\cdot\frac{3}{8}x^{\frac{3}{8}-1}$

$=4x^{\frac{1}{3}}+\frac{15}{8}x^{-\frac{5}{8}}\left(=4\sqrt[3]{x}+\frac{15}{8\sqrt[8]{x^5}}\right)$

(5) $y=x\sqrt{x}+\frac{3}{x\sqrt[3]{x^2}}$

$y=x\sqrt{x}+\frac{3}{x\sqrt[3]{x^2}}=x^{\frac{3}{2}}+3x^{-\frac{5}{3}}$

よって　$y'=\frac{3}{2}x^{\frac{3}{2}-1}+3\left(-\frac{5}{3}x^{-\frac{5}{3}-1}\right)$

$=\frac{3}{2}x^{\frac{1}{2}}-5x^{-\frac{8}{3}}$

$=\frac{3}{2}\sqrt{x}-\frac{5}{x^2\cdot\sqrt[3]{x^2}}$

6 ●次の関数を微分せよ。

(1) $y=\sqrt{x^2-4x+5}$

$y=\sqrt{x^2-4x+5}=(x^2-4x+5)^{\frac{1}{2}}$

よって　$y'=\frac{1}{2}(x^2-4x+5)^{\frac{1}{2}-1}(x^2-4x+5)'=\frac{1}{2}(x^2-4x+5)^{-\frac{1}{2}}(2x-4)$

$=\frac{x-2}{\sqrt{x^2-4x+5}}$

(2) $y=\frac{1}{\sqrt{1-x^2}}$

$y=\frac{1}{\sqrt{1-x^2}}=(1-x^2)^{-\frac{1}{2}}$

よって　$y'=-\frac{1}{2}(1-x^2)^{-\frac{1}{2}-1}(1-x^2)'=-\frac{1}{2}(1-x^2)^{-\frac{3}{2}}(-2x)$

$=\frac{x}{\sqrt{(1-x^2)^3}}\left(=\frac{x}{(1-x^2)\sqrt{1-x^2}}\right)$

別解　$y'=-\frac{(\sqrt{1-x^2})'}{(\sqrt{1-x^2})^2}=-\frac{\frac{-2x}{2\sqrt{1-x^2}}}{1-x^2}=\frac{x}{(1-x^2)\sqrt{1-x^2}}$

(3) $y=\sqrt[4]{x^2-2}+\sqrt[3]{2x+3}$

$y=\sqrt[4]{x^2-2}+\sqrt[3]{2x+3}=(x^2-2)^{\frac{1}{4}}+(2x+3)^{\frac{1}{3}}$

よって　$y'=\frac{1}{4}(x^2-2)^{\frac{1}{4}-1}(x^2-2)'+\frac{1}{3}(2x+3)^{\frac{1}{3}-1}(2x+3)'$

$=\frac{1}{4}(x^2-2)^{-\frac{3}{4}}\cdot2x+\frac{1}{3}(2x+3)^{-\frac{2}{3}}\cdot2$

$=\frac{x}{2\sqrt[4]{(x^2-2)^3}}+\frac{2}{3\sqrt[3]{(2x+3)^2}}$

(4) $y=x\sqrt{2x^2+1}$

$y'=(x)'\sqrt{2x^2+1}+x(\sqrt{2x^2+1})'=1\cdot\sqrt{2x^2+1}+x\cdot\frac{4x}{2\sqrt{2x^2+1}}$

$=\sqrt{2x^2+1}+\frac{2x^2}{\sqrt{2x^2+1}}=\frac{(2x^2+1)+2x^2}{\sqrt{2x^2+1}}=\frac{4x^2+1}{\sqrt{2x^2+1}}$

(5) $y=-\frac{4x}{\sqrt{x^2+3}}$

$y'=-\frac{(4x)'\sqrt{x^2+3}-4x(\sqrt{x^2+3})'}{(\sqrt{x^2+3})^2}=-\frac{4\sqrt{x^2+3}-4x\cdot\frac{2x}{2\sqrt{x^2+3}}}{x^2+3}$

$=-\frac{4(x^2+3)-4x^2}{(x^2+3)\sqrt{x^2+3}}=-\frac{12}{(x^2+3)\sqrt{x^2+3}}$

7 ●次の関数を微分せよ。

(1) $y=\sqrt[3]{x^3+2x}$

$y=\sqrt[3]{x^3+2x}=(x^3+2x)^{\frac{1}{3}}$

よって　$y'=\frac{1}{3}(x^3+2x)^{\frac{1}{3}-1}(x^3+2x)'=\frac{1}{3}(x^3+2x)^{-\frac{2}{3}}(3x^2+2)$

$=\frac{3x^2+2}{3\sqrt[3]{(x^3+2x)^2}}$

(2) $y=-\frac{1}{\sqrt[3]{3x^2+1}}$

$y=-\frac{1}{\sqrt[3]{3x^2+1}}=-(3x^2+1)^{-\frac{1}{3}}$

よって　$y'=-\left\{-\frac{1}{3}(3x^2+1)^{-\frac{1}{3}-1}(3x^2+1)'\right\}=\frac{1}{3}(3x^2+1)^{-\frac{4}{3}}\cdot6x$

$=\frac{2x}{\sqrt[3]{(3x^2+1)^4}}\left(=\frac{2x}{(3x^2+1)\sqrt[3]{3x^2+1}}\right)$

別解　$y'=\frac{(\sqrt[3]{3x^2+1})'}{(\sqrt[3]{3x^2+1})^2}=\frac{\frac{1}{3}\cdot\frac{1}{\sqrt[3]{(3x^2+1)^2}}\cdot6x}{\sqrt[3]{(3x^2+1)^2}}=\frac{2x}{\sqrt[3]{(3x^2+1)^4}}$

(3) $y=\sqrt{4x+1}-\sqrt[4]{5x^2-2}$

$y=\sqrt{4x+1}-\sqrt[4]{5x^2-2}=(4x+1)^{\frac{1}{2}}-(5x^2-2)^{\frac{1}{4}}$

よって　$y'=\frac{1}{2}(4x+1)^{\frac{1}{2}-1}(4x+1)'-\frac{1}{4}(5x^2-2)^{\frac{1}{4}-1}(5x^2-2)'$

$=\frac{1}{2}(4x+1)^{-\frac{1}{2}}\cdot4-\frac{1}{4}(5x^2-2)^{-\frac{3}{4}}\cdot10x$

$=\frac{2}{\sqrt{4x+1}}-\frac{5x}{2\sqrt[4]{(5x^2-2)^3}}$

(4) $y=(x+1)\sqrt{x^2-5}$

$y'=(x+1)'\sqrt{x^2-5}+(x+1)(\sqrt{x^2-5})'=1\cdot\sqrt{x^2-5}+(x+1)\frac{2x}{2\sqrt{x^2-5}}$

$=\sqrt{x^2-5}+\frac{x(x+1)}{\sqrt{x^2-5}}=\frac{(x^2-5)+x(x+1)}{\sqrt{x^2-5}}=\frac{2x^2+x-5}{\sqrt{x^2-5}}$

(5) $y=\frac{x^2}{\sqrt{x^2-4}}$

$y'=\frac{(x^2)'\sqrt{x^2-4}-x^2(\sqrt{x^2-4})'}{(\sqrt{x^2-4})^2}=\frac{2x\sqrt{x^2-4}-x^2\cdot\frac{2x}{2\sqrt{x^2-4}}}{x^2-4}$

$=\frac{2x(x^2-4)-x^3}{(x^2-4)\sqrt{x^2-4}}=\frac{x(x^2-8)}{(x^2-4)\sqrt{x^2-4}}$