

1 ●次の関数を微分せよ。

(1)  $y = x^5$   
 $y' = 5x^4$

(2)  $y = -2x^4$   
 $y' = -2 \cdot 4x^3 = -8x^3$

(3)  $y = x^3 - 3x^2$   
 $y' = 3x^2 - 3 \cdot 2x$   
 $= 3x^2 - 6x$

(4)  $y = 4x^3 + 5x^2 + 7x + 1$   
 $y' = 4 \cdot 3x^2 + 5 \cdot 2x + 7$   
 $= 12x^2 + 10x + 7$

(5)  $y = x^{-2}$   
 $y' = -2x^{-2-1} = -2x^{-3} \left( = -\frac{2}{x^3} \right)$

(6)  $y = -3x^{-3}$   
 $y' = -3(-3x^{-3-1}) = 9x^{-4} \left( = \frac{9}{x^4} \right)$

(7)  $y = \frac{1}{x^4}$   
 $\frac{1}{x^4} = x^{-4}$  であるから  
 $y' = -4x^{-4-1} = -4x^{-5} = -\frac{4}{x^5}$

(8)  $y = x^2 + \frac{1}{x^5}$   
 $x^2 + \frac{1}{x^5} = x^2 + x^{-5}$  であるから  
 $y' = 2x - 5x^{-5-1}$   
 $= 2x - 5x^{-6} = 2x - \frac{5}{x^6}$

2 ●次の関数を微分せよ。

(1)  $y = (x+2)(3x+4)$   
 $y' = (x+2)'(3x+4) + (x+2)(3x+4)'$   
 $= 1 \cdot (3x+4) + (x+2) \cdot 3$   
 $= 3x+4+3x+6$   
 $= 6x+10$

(2)  $y = (2x+3)(x^2-3)$   
 $y' = (2x+3)'(x^2-3) + (2x+3)(x^2-3)'$   
 $= 2 \cdot (x^2-3) + (2x+3) \cdot 2x$   
 $= 2x^2-6+4x^2+6x$   
 $= 6x^2+6x-6$

(3)  $y = x^2(x^2-2x)$   
 $y' = (x^2)'(x^2-2x) + x^2(x^2-2x)'$   
 $= 2x(x^2-2x) + x^2(2x-2)$   
 $= 2x^3-4x^2+2x^3-2x^2$   
 $= 4x^3-6x^2$

(4)  $y = (x^3-2x)x^4$   
 $y' = (x^3-2x)'x^4 + (x^3-2x)(x^4)'$   
 $= (3x^2-2)x^4 + (x^3-2x) \cdot 4x^3$   
 $= 3x^6-2x^4+4x^6-8x^4$   
 $= 7x^6-10x^4$

(5)  $y = (x^2+2x)(2x^3-x)$   
 $y' = (x^2+2x)'(2x^3-x) + (x^2+2x)(2x^3-x)'$   
 $= (2x+2)(2x^3-x) + (x^2+2x)(6x^2-1)$   
 $= 4x^4-2x^2+4x^3-2x+6x^4-x^2$   
 $+ 12x^3-2x$   
 $= 10x^4+16x^3-3x^2-4x$

(6)  $y = (3x^2-1)(2x^2+x)$   
 $y' = (3x^2-1)'(2x^2+x) + (3x^2-1)(2x^2+x)'$   
 $= 6x(2x^2+x) + (3x^2-1)(4x+1)$   
 $= 12x^3+6x^2+12x^3+3x^2-4x-1$   
 $= 24x^3+9x^2-4x-1$

3 ●次の関数を微分せよ。

(1)  $y = (3x-1)^2$   
 $y' = 2(3x-1)(3x-1)'$   
 $= 2(3x-1) \cdot 3$   
 $= 6(3x-1)$

(2)  $y = -(2x^2+x)^3$   
 $y' = -3(2x^2+x)^2(2x^2+x)'$   
 $= -3(2x^2+x)^2(4x+1)$

(3)  $y = (x+1)(2x+1)^2$   
 $y' = (x+1)'(2x+1)^2 + (x+1)((2x+1)^2)'$   
 $= (2x+1)^2 + (x+1) \cdot 2(2x+1)(2x+1)'$   
 $= (2x+1)^2 + (x+1) \cdot 2(2x+1) \cdot 2$   
 $= (2x+1)(2x+1) + 4(x+1)$   
 $= (2x+1)(6x+5)$

(4)  $y = \frac{1}{(x-2)^2}$   
 $\frac{1}{(x-2)^2} = (x-2)^{-2}$  であるから  
 $y' = -2(x-2)^{-3}(x-2)'$   
 $= -2(x-2)^{-3} \cdot 1$   
 $= -\frac{2}{(x-2)^3}$

(5)  $y = \frac{1}{(x^2-5x+2)^3}$   
 $\frac{1}{(x^2-5x+2)^3} = (x^2-5x+2)^{-3}$  であるから  
 $y' = -3(x^2-5x+2)^{-4}(x^2-5x+2)'$   
 $= -3(x^2-5x+2)^{-4}(2x-5)$   
 $= -\frac{3(2x-5)}{(x^2-5x+2)^4}$

(6)  $y = \frac{1}{(x^3+x)^4}$   
 $\frac{1}{(x^3+x)^4} = (x^3+x)^{-4}$  であるから  
 $y' = -4(x^3+x)^{-5}(x^3+x)'$   
 $= -4 \cdot \frac{3x^2+1}{(x^3+x)^5}$   
 $= -\frac{4(3x^2+1)}{(x^3+x)^5}$

4 ●次の関数を微分せよ。

(1)  $y = x^{\frac{3}{4}}$   
 $y' = \frac{3}{4}x^{\frac{3}{4}-1} = \frac{3}{4}x^{-\frac{1}{4}} \left( = \frac{3}{4\sqrt[4]{x}} \right)$

(2)  $y = \sqrt{x^3}$   
 $\sqrt{x^3} = x^{\frac{3}{2}}$  であるから  
 $y' = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

(3)  $y = \frac{1}{\sqrt[3]{x}}$   
 $\frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$  であるから  
 $y' = -\frac{1}{3}x^{-\frac{1}{3}-1} = -\frac{1}{3}x^{-\frac{4}{3}} = -\frac{1}{3x\sqrt[3]{x}}$

(4)  $y = x\sqrt[3]{x}$   
 $x\sqrt[3]{x} = x^{\frac{4}{3}}$  であるから  
 $y' = \frac{4}{3}x^{\frac{4}{3}-1} = \frac{4}{3}x^{\frac{1}{3}} = \frac{4}{3}\sqrt[3]{x}$

(5)  $y = 4x^{\frac{5}{4}} - 6x^{\frac{5}{3}}$   
 $y' = 4 \cdot \frac{5}{4}x^{\frac{5}{4}-1} - 6 \cdot \frac{5}{3}x^{\frac{5}{3}-1}$   
 $= 5x^{\frac{1}{4}} - 10x^{\frac{2}{3}}$

(6)  $y = 2x\sqrt{x} + \frac{1}{x\sqrt{x}}$   
 $2x\sqrt{x} + \frac{1}{x\sqrt{x}} = 2x^{\frac{3}{2}} + x^{-\frac{3}{2}}$  であるから  
 $y' = 2 \cdot \frac{3}{2}x^{\frac{3}{2}-1} - \frac{3}{2}x^{-\frac{3}{2}-1}$   
 $= 3x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{5}{2}}$   
 $= 3\sqrt{x} - \frac{3}{2x^2\sqrt{x}}$

5 ● 次の関数を微分せよ。

(1)  $y = \sqrt{2x+3}$

$$\begin{aligned} \sqrt{2x+3} &= (2x+3)^{\frac{1}{2}} \text{ であるから} \\ y' &= \frac{1}{2}(2x+3)^{\frac{1}{2}-1}(2x+3)' \\ &= \frac{1}{2}(2x+3)^{-\frac{1}{2}} \cdot 2 \\ &= \frac{1}{\sqrt{2x+3}} \end{aligned}$$

(3)  $y = \frac{1}{\sqrt{1-4x}}$

$$\begin{aligned} \frac{1}{\sqrt{1-4x}} &= (1-4x)^{-\frac{1}{2}} \text{ であるから} \\ y' &= -\frac{1}{2}(1-4x)^{-\frac{1}{2}-1}(1-4x)' \\ &= -\frac{1}{2}(1-4x)^{-\frac{3}{2}}(-4) \\ &= \frac{2}{\sqrt{(1-4x)^3}} \left( = \frac{2}{(1-4x)\sqrt{1-4x}} \right) \end{aligned}$$

(5)  $y = x\sqrt{x^2-4}$

$$\begin{aligned} y' &= (x)'\sqrt{x^2-4} + x(\sqrt{x^2-4})' \\ &= \sqrt{x^2-4} + x \cdot \frac{2x}{2\sqrt{x^2-4}} \\ &= \sqrt{x^2-4} + \frac{x^2}{\sqrt{x^2-4}} \\ &= \frac{2x^2-4}{\sqrt{x^2-4}} \end{aligned}$$

6 ● 次の関数を微分せよ。

(1)  $y = (2x+3)^2$

$$\begin{aligned} y' &= 2(2x+3)(2x+3)' \\ &= 2(2x+3) \cdot 2 \\ &= 4(2x+3) \\ &= 8x+12 \end{aligned}$$

(3)  $y = (x^2-2x)^3$

$$\begin{aligned} y' &= 3(x^2-2x)^2(x^2-2x)' \\ &= 3(x^2-2x)^2(2x-2) \\ &= 6(x^2-2x)^2(x-1) \\ &= 6x^2(x-2)^2(x-1) \end{aligned}$$

(5)  $y = (x^2+2x+3)^3$

$$\begin{aligned} y' &= 3(x^2+2x+3)^2(x^2+2x+3)' \\ &= 3(x^2+2x+3)^2(2x+2) \\ &= 6(x+1)(x^2+2x+3)^2 \end{aligned}$$

(2)  $y = \sqrt{x^2-2x}$

$$\begin{aligned} \sqrt{x^2-2x} &= (x^2-2x)^{\frac{1}{2}} \text{ であるから} \\ y' &= \frac{1}{2}(x^2-2x)^{\frac{1}{2}-1}(x^2-2x)' \\ &= \frac{1}{2}(x^2-2x)^{-\frac{1}{2}}(2x-2) \\ &= \frac{x-1}{\sqrt{x^2-2x}} \end{aligned}$$

(4)  $y = \sqrt[3]{x^2} + \sqrt[4]{4x+2}$

$$\begin{aligned} \sqrt[3]{x^2} + \sqrt[4]{4x+2} &= x^{\frac{2}{3}} + (4x+2)^{\frac{1}{4}} \text{ であるから} \\ y' &= \frac{2}{3}x^{\frac{2}{3}-1} + \frac{1}{4}(4x+2)^{\frac{1}{4}-1}(4x+2)' \\ &= \frac{2}{3}x^{-\frac{1}{3}} + \frac{1}{4}(4x+2)^{-\frac{3}{4}} \cdot 4 \\ &= \frac{2}{3\sqrt[3]{x}} + \frac{1}{\sqrt[4]{(4x+2)^3}} \end{aligned}$$

(6)  $y = \frac{x}{\sqrt{x^2+1}}$

$$\begin{aligned} y' &= \frac{(x)'\sqrt{x^2+1} - x(\sqrt{x^2+1})'}{(\sqrt{x^2+1})^2} \\ &= \frac{\sqrt{x^2+1} - x \cdot \frac{2x}{2\sqrt{x^2+1}}}{x^2+1} \\ &= \frac{x^2+1-x^2}{(x^2+1)\sqrt{x^2+1}} \\ &= \frac{1}{(x^2+1)\sqrt{x^2+1}} \end{aligned}$$

(2)  $y = (3x-4)^5$

$$\begin{aligned} y' &= 5(3x-4)^4(3x-4)' \\ &= 5(3x-4)^4 \cdot 3 \\ &= 15(3x-4)^4 \end{aligned}$$

(4)  $y = (x^2-3)^4$

$$\begin{aligned} y' &= 4(x^2-3)^3(x^2-3)' \\ &= 4(x^2-3)^3 \cdot 2x \\ &= 8x(x^2-3)^3 \end{aligned}$$

(6)  $y = \frac{1}{x^2-3x+1}$

$$\begin{aligned} \frac{1}{x^2-3x+1} &= (x^2-3x+1)^{-1} \text{ であるから} \\ y' &= -(x^2-3x+1)^{-2} \cdot (x^2-3x+1)' \\ &= -\frac{2x-3}{(x^2-3x+1)^2} \end{aligned}$$

7 ● 次の関数を微分せよ。

(1)  $y = x^{\frac{1}{4}}$

$$\begin{aligned} y' &= \frac{1}{4}x^{\frac{1}{4}-1} \\ &= \frac{1}{4}x^{-\frac{3}{4}} \end{aligned}$$

(3)  $y = \sqrt[3]{x}$

$$\begin{aligned} \sqrt[3]{x} &= x^{\frac{1}{3}} \text{ であるから} \\ y' &= \frac{1}{3}x^{\frac{1}{3}-1} \\ &= \frac{1}{3}x^{-\frac{2}{3}} \\ &= \frac{1}{3} \cdot \frac{1}{x^{\frac{2}{3}}} \\ &= \frac{1}{3\sqrt[3]{x^2}} \end{aligned}$$

(5)  $y = \sqrt[3]{x^2}$

$$\begin{aligned} \sqrt[3]{x^2} &= x^{\frac{2}{3}} \text{ であるから} \\ y' &= \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}} \\ &= \frac{2}{3} \cdot \frac{1}{x^{\frac{1}{3}}} = \frac{2}{3\sqrt[3]{x}} \end{aligned}$$

8 ● 次の関数を微分せよ。

(1)  $y = \sqrt{3x-2}$

$$\begin{aligned} \sqrt{3x-2} &= (3x-2)^{\frac{1}{2}} \text{ であるから} \\ y' &= \frac{1}{2}(3x-2)^{\frac{1}{2}-1} \cdot (3x-2)' \\ &= \frac{3}{2}(3x-2)^{-\frac{1}{2}} \\ &= \frac{3}{2\sqrt{3x-2}} \end{aligned}$$

(3)  $y = \frac{1}{\sqrt{4x-3}}$

$$\begin{aligned} \frac{1}{\sqrt{4x-3}} &= (4x-3)^{-\frac{1}{2}} \text{ であるから} \\ y' &= -\frac{1}{2}(4x-3)^{-\frac{1}{2}-1} \cdot (4x-3)' \\ &= -\frac{1}{2}(4x-3)^{-\frac{3}{2}} \cdot 4 \\ &= -\frac{2}{\sqrt{(4x-3)^3}} \\ &= -\frac{2}{(4x-3)\sqrt{4x-3}} \end{aligned}$$

(3)  $y = \frac{1}{\sqrt{4x-3}}$

$$\begin{aligned} \frac{1}{\sqrt{4x-3}} &= (4x-3)^{-\frac{1}{2}} \text{ であるから} \\ y' &= -\frac{1}{2}(4x-3)^{-\frac{1}{2}-1} \cdot (4x-3)' \\ &= -\frac{1}{2}(4x-3)^{-\frac{3}{2}} \cdot 4 \\ &= -\frac{2}{\sqrt{(4x-3)^3}} \\ &= -\frac{2}{(4x-3)\sqrt{4x-3}} \end{aligned}$$

(2)  $y = \frac{1}{x^3}$

$$\begin{aligned} \frac{1}{x^3} &= x^{-3} \text{ であるから} \\ y' &= -3x^{-3-1} \\ &= -3x^{-4} \\ &= -\frac{3}{x^4} \end{aligned}$$

(4)  $y = \frac{1}{\sqrt[3]{x}}$

$$\begin{aligned} \frac{1}{\sqrt[3]{x}} &= x^{-\frac{1}{3}} \text{ であるから} \\ y' &= -\frac{1}{3}x^{-\frac{1}{3}-1} \\ &= -\frac{1}{3}x^{-\frac{4}{3}} = -\frac{1}{3} \cdot \frac{1}{x^{\frac{4}{3}}} \\ &= -\frac{1}{3\sqrt[3]{x^4}} = -\frac{1}{3x\sqrt[3]{x}} \end{aligned}$$

(6)  $y = x\sqrt[3]{x}$

$$\begin{aligned} x\sqrt[3]{x} &= x^1 \cdot x^{\frac{1}{3}} = x^{1+\frac{1}{3}} = x^{\frac{4}{3}} \text{ であるから} \\ y' &= \frac{4}{3}x^{\frac{4}{3}-1} = \frac{4}{3}x^{\frac{1}{3}} \\ &= \frac{4}{3}\sqrt[3]{x} \end{aligned}$$

(2)  $y = \sqrt{x^2+3x}$

$$\begin{aligned} \sqrt{x^2+3x} &= (x^2+3x)^{\frac{1}{2}} \text{ であるから} \\ y' &= \frac{1}{2}(x^2+3x)^{\frac{1}{2}-1} \cdot (x^2+3x)' \\ &= \frac{1}{2}(x^2+3x)^{-\frac{1}{2}} \cdot (2x+3) \\ &= \frac{2x+3}{2\sqrt{x^2+3x}} \end{aligned}$$

(4)  $y = \frac{1}{\sqrt{x^2+2x}}$

$$\begin{aligned} \frac{1}{\sqrt{x^2+2x}} &= (x^2+2x)^{-\frac{1}{2}} \text{ であるから} \\ y' &= -\frac{1}{2}(x^2+2x)^{-\frac{1}{2}-1} \cdot (x^2+2x)' \\ &= -\frac{1}{2}(x^2+2x)^{-\frac{3}{2}} \cdot (2x+2) \\ &= -\frac{x+1}{\sqrt{(x^2+2x)^3}} \\ &= -\frac{x+1}{(x^2+2x)\sqrt{x^2+2x}} \end{aligned}$$