数学補習：講義プリント９回目

♣ 三角関数の導関数

定理.（三角関数の導関数） 次の公式が成り立つ。

[1]　$(\sin x)' = \cos x$

[2]　$(\cos x)' = -\sin x$

[3]　$(\tan x)' = \dfrac{1}{\cos^2 x}$

定理の証明.

(1)
$$\begin{aligned}(\sin x)' &= \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h} = \lim_{h\to 0}\frac{1}{h}\cdot 2\cos\left(x+\frac{h}{2}\right)\sin\frac{h}{2} = \lim_{h\to 0}\cos\left(x+\frac{h}{2}\right)\frac{\sin\frac{h}{2}}{\frac{h}{2}}\\ &= \cos x\end{aligned}$$

(2)
$$\begin{aligned}(\cos x)' &= \left\{\sin\left(x+\frac{\pi}{2}\right)\right\}' = \cos\left(x+\frac{\pi}{2}\right)\cdot\left(x+\frac{\pi}{2}\right)' = \cos\left(x+\frac{\pi}{2}\right)\\ &= -\sin x\end{aligned}$$

(3)
$$\begin{aligned}(\tan x)' &= \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)'\cos x-\sin x(\cos x)'}{\cos^2 x} = \frac{\cos^2 x+\sin^2 x}{\cos^2 x}\\ &= \frac{1}{\cos^2 x}\end{aligned}$$

例. 次の関数の導関数を求めよ。

(1) $y=\sin x\cos x$　　(2) $y=\sin(x^3-2)$

[解答] (1)
$$\begin{aligned}y' &= (\sin x)'\cos x+\sin x(\cos x)'\\ &= \cos x\cos x+\sin x(-\sin x) = \cos^2 x-\sin^2 x\end{aligned}$$

(2)
$$\begin{aligned}y' &= \left\{\sin(x^3-2)\right\}'\\ &= \cos(x^3-2)\cdot\left(x^3-2\right)' = \cos(x^3-2)\cdot\left(3x^2\right) = 3x^2\cos(x^3-2)\end{aligned}$$

問 1. 次の関数を微分せよ.

(1) $y = \sin(2x-3)$　(2) $y = \cos^5 x$　(3) $y = \tan 2x$

(4) $y = x\cos x$　(5) $y = \sin(3x^2-2)$　(6) $y = \dfrac{\sin x}{1+\cos x}$

(7) $y = x^2 \sin x$　(8) $y = \sqrt{1+\sin x}$　(9) $y = x\cos x - \sin x$

問 2. 次の関数を微分せよ.

(1) $y = 2x^4 - x^3 + x^2 + 1$

(2) $y = (2x^3 + 1)(3x^4 - 1)$

(3) $y = \dfrac{x^2 + 2x + 2}{x + 1}$

(4) $y = \sin 3x$

(5) $y = x \sin x + \cos x$

(6) $y = \cos^2 x$

(7) $y = \cos x^2$

(8) $y = \dfrac{1}{\tan x}$

(9) $y = \sin 3x \tan 2x$