

1 例 不定積分  $\int xe^{3x} dx$  を求めよ。

$$\begin{aligned} \int xe^{3x} dx &= \int x \left( \frac{e^{3x}}{3} \right)' dx = x \cdot \frac{e^{3x}}{3} - \int (x)' \frac{e^{3x}}{3} dx = \frac{1}{3} x e^{3x} - \frac{1}{3} \int e^{3x} dx \\ &= \frac{1}{3} x e^{3x} - \frac{1}{3} \cdot \frac{e^{3x}}{3} + C = \frac{1}{9} (3x-1)e^{3x} + C \quad (C \text{ は積分定数}) \end{aligned}$$

● 次の不定積分を求めよ。

(1)  $\int xe^{-2x} dx$

$$\begin{aligned} &\int xe^{-2x} dx \\ &= \int x \left( -\frac{e^{-2x}}{2} \right)' dx \\ &= x \left( -\frac{e^{-2x}}{2} \right) - \int (x)' \left( -\frac{e^{-2x}}{2} \right) dx \\ &= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x} dx \\ &= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \left( -\frac{e^{-2x}}{2} \right) + C \\ &= -\frac{1}{4} (2x+1)e^{-2x} + C \end{aligned}$$

(Cは積分定数, 以下同じ)

(2)  $\int x^3 \log x dx$

$$\begin{aligned} &\int x^3 \log x dx \\ &= \int \log x \left( \frac{x^4}{4} \right)' dx \\ &= \log x \cdot \left( \frac{x^4}{4} \right) - \int (\log x)' \cdot \frac{x^4}{4} dx \\ &= \frac{1}{4} x^4 \log x - \frac{1}{4} \int \frac{1}{x} \cdot x^4 dx \\ &= \frac{1}{4} x^4 \log x - \frac{1}{4} \int x^3 dx \\ &= \frac{1}{4} x^4 \log x - \frac{1}{4} \cdot \frac{x^4}{4} + C \\ &= \frac{1}{16} x^4 (4 \log x - 1) + C \end{aligned}$$

● 次の不定積分を求めよ。

(1)  $\int x \sin 3x dx$

$$\begin{aligned} &\int x \sin 3x dx \\ &= \int x \left( -\frac{\cos 3x}{3} \right)' dx \\ &= x \left( -\frac{\cos 3x}{3} \right) - \int (x)' \left( -\frac{\cos 3x}{3} \right) dx \\ &= -\frac{1}{3} x \cos 3x + \frac{1}{3} \int \cos 3x dx \\ &= -\frac{1}{3} x \cos 3x + \frac{1}{3} \cdot \frac{\sin 3x}{3} + C \\ &= -\frac{1}{9} (3x \cos 3x - \sin 3x) + C \end{aligned}$$

(2)  $\int \frac{x}{\cos^2 x} dx$

$$\begin{aligned} &\int \frac{x}{\cos^2 x} dx \\ &= \int x (\tan x)' dx \\ &= x \tan x - \int (x)' \tan x dx \\ &= x \tan x - \int \tan x dx \\ &= x \tan x - \int \frac{\sin x}{\cos x} dx \\ &= x \tan x + \int \frac{-\sin x}{\cos x} dx \\ &= x \tan x + \int \frac{(\cos x)'}{\cos x} dx \\ &= x \tan x + \log |\cos x| + C \end{aligned}$$

2 例 不定積分  $\int \log(x+2) dx$  を求めよ。

$$\begin{aligned} \int \log(x+2) dx &= \int (\log(x+2))(x+2)' dx = (\log(x+2))(x+2) - \int (\log(x+2))'(x+2) dx \\ &= (x+2) \log(x+2) - \int \frac{1}{x+2} \cdot (x+2) dx = (x+2) \log(x+2) - x + C \end{aligned}$$

(Cは積分定数)

● 次の不定積分を求めよ。

(1)  $\int \log(x-5) dx$

$$\begin{aligned} &\int \log(x-5) dx \\ &= \int (\log(x-5))(x-5)' dx = (\log(x-5))(x-5) - \int (\log(x-5))'(x-5) dx \\ &= (x-5) \log(x-5) - \int \frac{1}{x-5} \cdot (x-5) dx = (x-5) \log(x-5) - \int dx \\ &= (x-5) \log(x-5) - x + C \end{aligned}$$

(2)  $\int \log(1-4x) dx$

$$\begin{aligned} &\int \log(1-4x) dx \\ &= \int (\log(1-4x)) \left( \frac{1-4x}{-4} \right)' dx = (\log(1-4x)) \cdot \frac{1-4x}{-4} - \int (\log(1-4x))' \cdot \frac{1-4x}{-4} dx \\ &= \frac{1}{4} (4x-1) \log(1-4x) - \int \frac{-4}{1-4x} \cdot \frac{1-4x}{-4} dx = \frac{1}{4} (4x-1) \log(1-4x) - \int dx \\ &= \frac{1}{4} (4x-1) \log(1-4x) - x + C \end{aligned}$$

3 ● 次の不定積分を求めよ。

(1)  $\int xe^x dx$

$$\begin{aligned} (\text{与式}) &= \int x(e^x)' dx = x e^x - \int e^x dx \\ &= x e^x - e^x + C = (x-1)e^x + C \quad (C \text{ は積分定数, 以下同様}) \end{aligned}$$

(2)  $\int (x-1) \sin x dx$

$$\begin{aligned} (\text{与式}) &= \int (x-1)(-\cos x)' dx \\ &= -(x-1) \cos x + \int \cos x dx \\ &= -(x-1) \cos x + \sin x + C \end{aligned}$$

(3)  $\int (3x-1)e^{-x} dx$

$$\begin{aligned} (\text{与式}) &= \int (3x-1)(-e^{-x})' dx \\ &= -(3x-1)e^{-x} + \int 3e^{-x} dx \\ &= -(3x-1)e^{-x} - 3e^{-x} + C \\ &= -(3x+2)e^{-x} + C \end{aligned}$$

(4)  $\int \log(x+2) dx$

$$\begin{aligned} (\text{与式}) &= \int (x+2) \log(x+2) dx \\ &= (x+2) \log(x+2) - \int (x+2) \cdot \frac{1}{x+2} dx \\ &= (x+2) \log(x+2) - \int dx \\ &= (x+2) \log(x+2) - x + C \end{aligned}$$

(5)  $\int \frac{\log x}{x^3} dx$

$$\begin{aligned} (\text{与式}) &= \int \left( -\frac{1}{2x^2} \right)' \log x dx \\ &= -\frac{1}{2x^2} \log x + \int \frac{1}{2x^2} \cdot \frac{1}{x} dx \\ &= -\frac{1}{2x^2} \log x + \frac{1}{2} \int \frac{1}{x^3} dx \\ &= -\frac{1}{2x^2} \log x - \frac{1}{4x^2} + C \\ &= -\frac{1}{4x^2} (2 \log x + 1) + C \end{aligned}$$

4 ● 次の不定積分を求めよ。

(1)  $\int x \cos x dx$

$$\begin{aligned} (\text{与式}) &= \int x(\sin x)' dx = x \sin x - \int \sin x dx \\ &= x \sin x + \cos x + C \quad (C \text{ は積分定数, 以下同様}) \end{aligned}$$

(2)  $\int (x+3)e^x dx$

$$\begin{aligned} (\text{与式}) &= \int (x+3)(e^x)' dx = (x+3)e^x - \int e^x dx \\ &= (x+3)e^x - e^x + C = (x+2)e^x + C \end{aligned}$$

(3)  $\int (2x-1) \sin 3x dx$

$$\begin{aligned} (\text{与式}) &= \int (2x-1) \left( -\frac{1}{3} \cos 3x \right)' dx \\ &= -\frac{1}{3} (2x-1) \cos 3x + \int 2 \cdot \frac{1}{3} \cos 3x dx \\ &= -\frac{1}{3} (2x-1) \cos 3x + \frac{2}{9} \sin 3x + C \end{aligned}$$

(4)  $\int \log(x-3) dx$

$$\begin{aligned} (\text{与式}) &= \int (x-3) \log(x-3) dx = (x-3) \log(x-3) - \int (x-3) \cdot \frac{1}{x-3} dx \\ &= (x-3) \log(x-3) - \int dx = (x-3) \log(x-3) - x + C \end{aligned}$$

(5)  $\int \frac{\log x}{x^4} dx$

$$\begin{aligned} (\text{与式}) &= \int \left( -\frac{1}{3x^3} \right)' \log x dx = -\frac{1}{3x^3} \log x + \int \frac{1}{3x^3} \cdot \frac{1}{x} dx \\ &= -\frac{1}{3x^3} \log x + \frac{1}{3} \int \frac{1}{x^4} dx = -\frac{1}{3x^3} \log x - \frac{1}{9x^3} + C \\ &= -\frac{1}{9x^3} (3 \log x + 1) + C \end{aligned}$$

5 ● 次の不定積分を求めよ。

(1)  $\int (2x-1)e^{3x} dx$

$$\begin{aligned} \text{(与式)} &= \int (2x-1) \left(\frac{1}{3}e^{3x}\right)' dx \\ &= \frac{1}{3}(2x-1)e^{3x} - \int 2 \cdot \frac{1}{3}e^{3x} dx \\ &= \frac{1}{3}(2x-1)e^{3x} - \frac{2}{9}e^{3x} + C \\ &= \frac{1}{9}(6x-5)e^{3x} + C \quad (C \text{ は積分定数, 以下同様}) \end{aligned}$$

(2)  $\int (4x-1)\cos 2x dx$

$$\begin{aligned} \text{(与式)} &= \int (4x-1) \left(\frac{1}{2}\sin 2x\right)' dx \\ &= \frac{1}{2}(4x-1)\sin 2x - \int 4 \cdot \frac{1}{2}\sin 2x dx \\ &= \frac{1}{2}(4x-1)\sin 2x + \cos 2x + C \end{aligned}$$

(3)  $\int \log(3x+5) dx$

$$\begin{aligned} \text{(与式)} &= \int \left(\frac{1}{3}(3x+5)\right)' \log(3x+5) dx \\ &= \frac{1}{3}(3x+5)\log(3x+5) - \int \frac{1}{3}(3x+5) \cdot \frac{3}{3x+5} dx \\ &= \frac{1}{3}(3x+5)\log(3x+5) - \int dx \\ &= \frac{1}{3}(3x+5)\log(3x+5) - x + C \end{aligned}$$

(4)  $\int x \log(x^2+3) dx$

$$\begin{aligned} \text{(与式)} &= \int \left(\frac{1}{2}(x^2+3)\right)' \log(x^2+3) dx \\ &= \frac{1}{2}(x^2+3)\log(x^2+3) - \int \frac{1}{2}(x^2+3) \cdot \frac{2x}{x^2+3} dx \\ &= \frac{1}{2}(x^2+3)\log(x^2+3) - \int x dx \\ &= \frac{1}{2}(x^2+3)\log(x^2+3) - \frac{1}{2}x^2 + C \end{aligned}$$

6 ● 次の不定積分を求めよ。

(1)  $\int (3x-1)\sin 2x dx$

$$\begin{aligned} \text{(与式)} &= \int (3x-1) \left(-\frac{1}{2}\cos 2x\right)' dx \\ &= -\frac{1}{2}(3x-1)\cos 2x + \int 3 \cdot \frac{1}{2}\cos 2x dx \\ &= -\frac{1}{2}(3x-1)\cos 2x + \frac{3}{4}\sin 2x + C \\ &\quad (C \text{ は積分定数, 以下同様}) \end{aligned}$$

(2)  $\int (6x+5)e^{2x} dx$

$$\begin{aligned} \text{(与式)} &= \int (6x+5) \left(\frac{1}{2}e^{2x}\right)' dx \\ &= \frac{1}{2}(6x+5)e^{2x} - \int 6 \cdot \frac{1}{2}e^{2x} dx \\ &= \frac{1}{2}(6x+5)e^{2x} - \frac{3}{2}e^{2x} + C \\ &= (3x+1)e^{2x} + C \end{aligned}$$

(3)  $\int \log(4x-1) dx$

$$\begin{aligned} \text{(与式)} &= \int \left(\frac{1}{4}(4x-1)\right)' \log(4x-1) dx \\ &= \frac{1}{4}(4x-1)\log(4x-1) - \int \frac{1}{4}(4x-1) \cdot \frac{4}{4x-1} dx \\ &= \frac{1}{4}(4x-1)\log(4x-1) - \int dx \\ &= \frac{1}{4}(4x-1)\log(4x-1) - x + C \end{aligned}$$

(4)  $\int e^x \log(e^x+2) dx$

$$\begin{aligned} \text{(与式)} &= \int (e^x+2)' \log(e^x+2) dx \\ &= (e^x+2)\log(e^x+2) - \int (e^x+2) \cdot \frac{e^x}{e^x+2} dx \\ &= (e^x+2)\log(e^x+2) - \int e^x dx \\ &= (e^x+2)\log(e^x+2) - e^x + C \end{aligned}$$

7 ● 次の不定積分を求めよ。

(1)  $\int x^2 e^{-x} dx$

$$\begin{aligned} \text{(与式)} &= \int x^2(-e^{-x})' dx = -x^2 e^{-x} + \int 2x e^{-x} dx \\ &= -x^2 e^{-x} + 2 \int x(-e^{-x})' dx = -x^2 e^{-x} - 2x e^{-x} + 2 \int e^{-x} dx \\ &= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C = -(x^2+2x+2)e^{-x} + C \\ &\quad (C \text{ は積分定数, 以下同様}) \end{aligned}$$

(2)  $\int (\log x)^2 dx$

$$\begin{aligned} \text{(与式)} &= \int (x)'(\log x)^2 dx = x(\log x)^2 - \int x \cdot 2\log x \cdot \frac{1}{x} dx \\ &= x(\log x)^2 - 2 \int (x)' \log x dx = x(\log x)^2 - 2x \log x + 2 \int x \cdot \frac{1}{x} dx \\ &= x(\log x)^2 - 2x \log x + 2 \int dx = x(\log x)^2 - 2x \log x + 2x + C \end{aligned}$$

(3)  $\int e^x \sin x dx$

$$\begin{aligned} \text{(与式)} &= \int (e^x)' \sin x dx = e^x \sin x - \int e^x \cos x dx \\ &= e^x \sin x - \int (e^x)' \cos x dx = e^x \sin x - e^x \cos x + \int e^x(-\sin x) dx \\ &= e^x \sin x - e^x \cos x - \int e^x \sin x dx \end{aligned}$$

ゆえに  $\int e^x \sin x dx = \frac{1}{2}e^x(\sin x - \cos x) + C$

【別解】  $(e^x \sin x)' = e^x \sin x + e^x \cos x \dots\dots ①$

$(e^x \cos x)' = e^x \cos x - e^x \sin x \dots\dots ②$

①-② より  $(e^x \sin x - e^x \cos x)' = 2e^x \sin x$

ゆえに  $\int e^x \sin x dx = \frac{1}{2}e^x(\sin x - \cos x) + C$

8 ● 次の不定積分を求めよ。

(1)  $\int x(\log x)^2 dx$

$$\begin{aligned} \text{(与式)} &= \int \left(\frac{1}{2}x^2\right)' (\log x)^2 dx = \frac{1}{2}x^2(\log x)^2 - \int \frac{1}{2}x^2 \cdot 2\log x \cdot \frac{1}{x} dx \\ &= \frac{1}{2}x^2(\log x)^2 - \int x \log x dx = \frac{1}{2}x^2(\log x)^2 - \int \left(\frac{1}{2}x^2\right)' \log x dx \\ &= \frac{1}{2}x^2(\log x)^2 - \frac{1}{2}x^2 \log x + \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\ &= \frac{1}{2}x^2(\log x)^2 - \frac{1}{2}x^2 \log x + \frac{1}{2} \int x dx \\ &= \frac{1}{2}x^2(\log x)^2 - \frac{1}{2}x^2 \log x + \frac{1}{4}x^2 + C \end{aligned}$$

(2)  $\int x^2 \sin x dx$

$$\begin{aligned} \text{(与式)} &= \int x^2(-\cos x)' dx = -x^2 \cos x + \int 2x \cos x dx \\ &= -x^2 \cos x + 2 \int x(\sin x)' dx = -x^2 \cos x + 2x \sin x - 2 \int \sin x dx \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x + C \end{aligned}$$

(3)  $\int e^x \cos x dx$

$$\begin{aligned} \text{(与式)} &= \int (e^x)' \cos x dx = e^x \cos x - \int e^x(-\sin x) dx \\ &= e^x \cos x + \int (e^x)' \sin x dx = e^x \cos x + e^x \sin x - \int e^x \cos x dx \end{aligned}$$

ゆえに  $\int e^x \cos x dx = \frac{1}{2}e^x(\sin x + \cos x) + C$

【別解】  $(e^x \sin x)' = e^x \sin x + e^x \cos x \dots\dots ①$

$(e^x \cos x)' = e^x \cos x - e^x \sin x \dots\dots ②$

①+② より  $(e^x \sin x + e^x \cos x)' = 2e^x \cos x$

ゆえに  $\int e^x \cos x dx = \frac{1}{2}e^x(\sin x + \cos x) + C$